Consumer demand systems and aggregate consumption in the USA: an application of the extended linear expenditure system

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Consumer demand systems and aggregate consumption in the USA: *an application of the extended linear expenditure system.* Two complete demand systems are fitted to USA data for the period 1930–72, using ML methods. The well known linear expenditure system (LES) yields γ-estimates that are very sensitive to error specification. The extended linear expenditure system (ELES), with disposable income and prices as explanatory variables, yields more stable γ-estimates. Both sets of estimates roughly coincide at a maximum of the likelihood function for each model. In addition, ELES equations add up to a 'Keynesian' consumption function. An estimate of the MPC is available and the saving effect of relative price changes can be measured.

Systèmes de demande du consommateur et fonction de demande globale aux Etats-Unis: une application du système linéaire de dépense développé. L'article contient deux systèmes complets de demande ajustés aux données américaines de la période 1930–72. Le système linéaire de dépense (LES) bien connu donne des estimations de γ (le paramètre de subsistance) qui sont très sensibles à la caractérisation de l'erreur. Le système linéaire de dépense développé (ELES), qui comprend le revenu disponible et les prix comme variables explicatives, donne des estimations plus stables de γ. Les deux groupes d'estimations sont semblables au point maximum de la fonction de vraisemblance de chaque modèle. De plus, les équations du système ELES correspondent à une fonction de consommation keynesienne. Une estimation de la propension marginale à consommer a été faite, et il est possible de calculer l'effet d'épargne de changements des prix relatifs. On postule que chaque système comporte la structure de retards la plus simple possible: un coefficient d'autocorrélation du premier degré commun à toutes les équations. Au maximum de la fonction de vraisemblance, le coefficient estimé est 0.85 pour les deux systèmes (LES et ELES). Même s'il semble que l'autocorrélation soit éliminée de cette façon, il sera nécessaire de poursuivre le travail aux plans théorique et empirique pour relier le système LES et les estimations des travaux sur les fonctions dynamiques de

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demande. On devrait trouver une interprétation économique des coefficients d'autocorrélation reliés spécifiquement à des marchandises. De plus, la fonction de dépense globale devrait être compatible avec les résultats d'une grande variété de travaux post-keynesiens sur le sujet.

I / INTRODUCTION

Consumer demand systems and aggregate consumption functions are treated separately in applied econometrics. In the expenditure allocation problem underlying all demand systems, models of increased generality are used to estimate price and expenditure responses and test the implications of demand theory under the assumption that total consumption expenditure is exogenously given. In the intertemporal consumer problem underlying any treatment of savings, total consumption expenditure is endogenized, and wealth-related concepts are introduced as basic determinants of behaviour. Such concepts are kept apart from the expenditure allocation problem, and relative prices are ignored as determinants of savings.

The rationale most often used to keep these areas apart is that two assumptions (intertemporal additivity and certain, stationary price expectations) are sufficient to break the intertemporal consumer problem into two parts: a sequence of one-period utility maximizations that solves the expenditure allocation problem once total consumption expenditure is known, and the determination of the optimum savings program itself. It should be apparent, though, that such a rationale is not sufficient in applied work: it is based on economic theory, and it ignores the consequences of alternative stochastic specifications for estimates of the *same* parameter set.

The purpose of this paper is to examine such consequences in a particular case, widely used in applied demand theory. Intertemporal utility maximization with a fixed subjective discount rate, stationary price expectations, and a Klein-Rubin utility function yields the extended linear expenditure system (ELES) with income and prices as explanatory variables (see Lluch, 1973). As expected, ELES can be decomposed into the linear expenditure system LES, derived on the assumption of instantaneous utility maximization (see Goldberger, 1967; Brown and Deaton, 1972), and an aggregate consumption function which is the sum of the commodity expenditure equations. The parameter set is identical in LES and ELES, except for the presence in ELES of the Keynesian MPC.

There are two alternatives in estimation. If intertemporal utility maximization is the postulated hypothesis, then ELES should be stochastically specified (even if its deterministic part can be decomposed into LES and an aggregate consumption function). If, instead, instantaneous utility maximization is the postulated hypothesis, then LES should be stochastically specified, (yielding the conventional singular covariance matrix). The point at issue is whether the two procedures yield different estimates of the same parameter set and under what conditions they coincide – as indicated by the theory.

It will be shown that estimates may vary widely in the particular case chosen. The erratic behavior of γ-estimates (the 'subsistence' parameters) in the LES literature is well known. This paper documents their extreme sensitivity to assumptions regarding first-order autocorrelation in the error terms. ELES estimates are much less sensitive. Both sets of γ-estimates are, however, roughly the same when the value of the first-order autocorrelation coefficient is such that the likelihood function is maximized in each model.

The results reported in the paper may be of interest in a broader context. The ELES model is the first attempt to use income, instead of total expenditure, as an explanatory variable in a complete system of demand equations derived from a utility maximization hypothesis. The door is open for fuller integrations of the aggregate consumption and demand literatures. Permanent income considerations remain to be incorporated into demand theory, and they seem an important ingredient in a better treatment of demand for durable goods, for example. Similarly, the effect of relative price changes upon savings is a potentially important and unexplored problem.

The paper is organized as follows. In section II both LES and ELES are defined, together with their stochastic specifications. In section III maximum likelihood estimates, using USA data, are reported for both models. In section IV the ELES aggregate consumption function is compared with a naïve Keynesian function. The paper ends with a summary and suggestions for further work.

II / DEMAND SYSTEMS AND AGGREGATE CONSUMPTION

Deterministic models

Consider the extended linear expenditure system (ELES),

$$v_i = p_i\gamma_i + \beta_i^* \left(y - \sum_{j=1}^{n} p_j\gamma_j\right), \quad (i = 1, \ldots, n), \tag{1}$$

which expresses consumption expenditure $v_1, \ldots, v_n$ as a linear function of prices $p_1, \ldots, p_n$ and income y with fixed parameters that satisfy the constraints

$$\beta_i^* = \mu\beta_i, \quad 0 < \beta_i < 1, \quad \sum_{i=1}^{n} \beta_i = 1, \quad v_i - p_i\gamma_i > 0.$$

In Lluch (1973) ELES is derived from an intertemporal formulation of the consumer problem[1]. It represents the optimal allocation of expenditures at

1 Besides the assumptions of intertemporal additivity and static price expectations held with certainty made in the formulation of the problem, ELES as given in (1) incorporates the further assumption that the present value of expected changes in labour income is zero – so that permanent and measured income are the same.

the beginning of the consumer plan, when the instantaneous utility function is the Klein-Rubin (1947) indicator

$$u(q) = \beta' \log (q - \gamma), \tag{2}$$

where $q = (q_1, \ldots, q_n)'$, $q_i = v_i/p_i$, $\beta = (\beta_1, \ldots, \beta_n)'$, $\gamma = (\gamma_1, \ldots, \gamma_n)'$.

Adding up expenditures in (1) and using the restriction $\sum\beta_i = 1$, we obtain the aggregate consumption function associated with ELES,

$$v = (1 - \mu) \sum_{i=1}^{n} p_i\gamma_i + \mu y, \tag{3}$$

where $v = \sum_{i=1}^{n} v_i$. Thus, the parameter μ in ELES is the marginal propensity to consume MPC. In fact, equation (3) is a Keynesian consumption function, with the intercept defined as a linear function of prices.

In (3) we can express y as a function of v. Substituting this function into (1) we obtain

$$v_i = p_i\gamma_i + \beta_i \left(v - \sum_{j=1}^{n} p_j\gamma_j \right), \tag{4}$$

which is the widely used linear expenditure system LES, obtained by maximizing (2) under the constraint $\sum p_i q_i = v$. Thus, ELES can be decomposed into LES and the aggregate consumption function (3).

The properties and interpretation of LES are well known (see Goldberger, 1967; Goldberger and Gamaletsos, 1970). Only the elasticity formulas are given here. For all $i, j = 1, \ldots, n$ let us define the following: w_i, w_i^* are the average and "subsistence" budget shares v_i/v, $p_i\gamma_i/\sum p_i\gamma_i$, respectively; η_i is the expenditure demand elasticity; η_{ij}, η_{ij}^* are the uncompensated and expenditure compensated price elasticities; $-\phi$ is the "supernumerary" ratio $(v - \sum p_i\gamma_i)/v$.

The following elasticities are derived from (4) and (2) for all $i, j = 1, \ldots, n$:

$$\eta_i = \beta_i/w_i, \tag{5a}$$

$$\eta_{ij} = \begin{cases} \phi\eta_i - \eta_i w_i(1 + \phi\eta_i), & i = j, \\ \quad\;\; - \eta_i w_j(1 + \phi\eta_j) & i \neq j, \end{cases} \tag{5b}$$

$$\eta_{ij}^* = \begin{cases} \eta_i(1 - \beta_i)\phi, & i = j, \\ -\eta_i\beta_j\phi, & i \neq j. \end{cases} \tag{5c}$$

Equations (5) collect the expenditure and price responses in LES, system (4). Under the utility specification (2), the following inequalities hold: $\eta_{ii}, \eta_{ii}^* < 0$; $\eta_{ij} < 0$, $\eta_{ij}^* > 0$, for $i \neq j$.

The formal relationship between (5) and income and price responses in ELES, system (1), is easily established. Let $\tilde{\eta}_i$, $\tilde{\eta}_{ij}$, $\tilde{\eta}_{ij}^*$ denote income, and (uncompensated and income compensated) price elasticities in ELES. Also in ELES, let η, η^j be the elasticities of total consumption expenditures with respect to income and the j^{th} price, respectively. From (3) it follows that

$$\eta = \mu - (1 - \mu)\phi, \tag{6a}$$

$$\eta^j = (1 - \mu)(1 + \phi\eta_j)w_j. \tag{6b}$$

Notice that $\eta + \sum_j \eta^j = 1$. Also, it follows from (1) and (5) that[2]

$$\tilde{\eta}_i = \eta\eta_i, \tag{7a}$$

$$\tilde{\eta}_{ij} = \begin{cases} \mu\eta_{ii} + (1 - \mu)\phi\eta_i & i = j, \\ \mu\eta_{ij} & i \neq j, \end{cases} \tag{7b}$$

$$\tilde{\eta}_{ij}{}^* = \tilde{\eta}_{ij} + w_i\eta_j. \tag{7c}$$

Relations (6) and (7) justify centring attention on (5) only, as will be done in the rest of this paper.

Stochastic specification and estimation methods

Two models are estimated in this paper. The first is the conventional LES, which, under the assumption of no serial correlation in the errors, may be written as

$$\begin{aligned} v_{it} &= p_{it}\gamma_i + \beta_i\left(v_t - \sum_{i=1}^{n} p_{it}\gamma_i\right) + u_{it}, \\ E(u_t) &= 0, \\ E(u_t u_s') &= \begin{cases} \Lambda, & t = s, \\ 0, & t \neq s, \end{cases} \end{aligned} \tag{8}$$

where $i = 1, \ldots, n$; $s, t = 1, \ldots, T$; $u_t = (u_{1t}, \ldots, u_{nt})'$; and Λ is an $n \times n$ singular variance-covariance matrix of errors.[3] Furthermore, p_{it}, v_t are taken to be non-stochastic or, if stochastic, independent of u_{it}. Model (8) is the stochastic counterpart of (4), when the postulated consumer behaviour is based upon instantaneous utility maximization.

2 Expressions (7*a*) and (7*b*) result directly from algebraic manipulation of (1) and (3). Expression (7*c*) indicates that expenditure compensation in LES and income compensation in ELES are the same: $w_i\eta_j$. This is not immediately obvious: in general, the indirect utility *functional* (F) for the intertemporal formulation of the consumer problem differs from the indirect utility *function* (f) for the conventional formulation, and so does the Slutsky equation (see Lluch and Morishima, 1973, 175). But in this particular case, F is a linear transformation of f. The relevant terms in the differential of f for LES and ELES are identical, with dy (ELES) instead of with dv (LES).

3 Maximum likelihood methods of estimation, in spite of $|\Lambda| = 0$, are well known. The procedure is to omit one equation (results being invariant to the choice of equation to be omitted) and use the constraint $\Sigma\beta_i = 1$ to obtain the β-estimate for that equation (see Parks, 1971; Solari, 1969).

The second model is ELES, which, under the assumption of no serial correlation in the errors, may be written as

$$v_{it} = p_{it}\gamma_i + \beta_i^* \left(y_t - \sum_{i=1}^{n} p_{it}\gamma_i \right) + e_{it},$$
$$E(e_t) = 0, \tag{9}$$
$$E(e_t e_s') = \begin{cases} \Omega, & t = s, \\ 0, & t \neq s, \end{cases}$$

where $i = 1, \ldots, n$; $s, t = 1, \ldots, T$; $e_t = (e_{1t}, \ldots, e_{nt}')$; $\beta_i^* = \mu\beta_i$; and Ω is an $n \times n$ positive definite variance-covariance matrix of errors. Furthermore, p_{it}, y_t are taken to be non-stochastic or, if stochastic, independent of e_{it}. Model (9) is the stochastic counterpart of (1), which postulates consumer behaviour based upon intertemporal utility maximization. Summing the equations in (9) we obtain the stochastic equivalent of the aggregate consumption function (3):

$$v_t = (1 - \mu) \sum_{i=1}^{n} p_{it}\gamma_i + \mu y_t + \epsilon_t, \tag{10}$$

where

$$\epsilon_t = \sum_{i=1}^{n} e_{it}.$$

It then follows that the error term on (8), the stochastic counterpart of (4), is given by

$$u_{it} = e_{it} - \beta_i \sum_{i=1}^{n} e_{it}, \tag{11}$$

for all i, t. If (9) is the postulated model, equations (10) and (11) clearly imply contemporaneous correlation between u_{it} and the (stochastic) explanatory variable v_t, that is,

$$\text{plim } T^{-1} \sum_{t=1}^{T} v_t u_{it} \neq 0, \quad i = 1, \ldots, n. \tag{12}$$

Conventional ML methods of estimation of LES would not be appropriate in this case, that is, they would yield inconsistent parameter estimates.[4]

Models (8) and (9) represent clear alternatives in applied work, reflecting alternative postulates on consumer behaviour. In both models, non-zero covariances across equations are allowed for at each point of time, but serial correlation is ruled out. (Notice that the key result (12) holds if serial correlation is specified, and it would still be possible to obtain consistent parameter estimates of ELES.) However, sample estimates of the parameters of both

4 The biases involved in estimating (8), when the true model is (9), have been worked out in the context of cross-section estimation by Powell (1973).

models, and more particularly their standard errors, may be considerably biased by incorrect error specification. The approach adopted here is to assume a more complex error structure if the residuals obtained on estimating (8) and (9) suggest that such extensions are needed.

Let us assume a first-order autoregressive error structure for each model:

$$u_{it} = \rho u_{i,t-1} + u_{it}^*, \text{ (LES)},$$

$$e_{it} = \lambda e_{i,t-1} + e_{it}^*, \text{ (ELES)},$$

for all i, t, where ρ and λ are the first-order autocorrelation coefficients[5] and u_{it}^* and e_{it}^* are error terms which are contemporaneously correlated across equations in the respective models but uncorrelated over time. Then all the results in this section hold if the variables in the models are replaced by their first-order transforms, that is, if a variable x_t, say, is replaced by $x_t - \rho x_{t-1}$ in LES and by $x_t - \lambda x_{t-1}$ in ELES.

On the assumption that u_t and e_t (or u_t^* and e_t^*) come from multivariate normal distributions, maximum likelihood estimates of the parameters β_i, γ_i in (8) and β_i^*, γ_i in (9) (or the first-order transforms of (8) and (9)) have been obtained using the (modified) Gauss-Newton method with analytical derivatives.[6] The μ-estimate in (9) is obtained from the restriction $\mu = \sum \beta_i^*$.

Asymptotic estimates of the variance-covariance matrix of estimated parameters in (8) and (9) were obtained from the Hessian of the log-likelihood function (see, for example, Dhrymes, 1970, 36) evaluated at estimated parameter values. In (9), it is necessary to obtain a $(2n + 1)$ variance-covariance matrix for μ, β, γ. Let $\Sigma_{\beta^*\beta^*}$, $\Sigma_{\beta^*\gamma}$, $\Sigma_{\gamma\gamma}$ be the estimated variance-covariance matrices of order n for β^*, γ. It follows[7] that

$$\begin{bmatrix} \iota'\Sigma_{\beta^*\beta_\iota^*} & \mu^{-1}\iota'\Sigma_{\beta^*\beta^*}J' & \iota'\Sigma_{\beta^*\gamma} \\ & \mu^{-2}J\Sigma_{\beta^*\beta^*}J' & \mu^{-1}J\Sigma_{\beta^*\gamma} \\ & & \Sigma_{\gamma\gamma} \end{bmatrix},$$

is the $(2n + 1)$ variance-covariance matrix for μ, β, γ, where $J = (I - \beta\iota')$, I is an $n \times n$ unit matrix, and ι is the n-column vector of unit elements.

5 For simplicity, we exclude serial correlation across equations in each model. Berndt and Savin (forthcoming) have recently shown that in this case, the first-order autocorrelation coefficients for LES must be the same in each equation. In section III we present some empirical evidence for also dropping the equation subscript on λ.

6 The program used was written for IBM by Y. Bard. For a full description, see Bard (1967). For model (8), results have been compared with those obtained using LINEX, a LES specific program (see Solari, 1969; Carlevaro and Rossier 1970). Parameter estimates were the same to five significant figures. Standard errors were slightly smaller using Bard's program due to different specification of the Hessian (for a test run using 1929–71 data, the maximum difference was 2.1 per cent).

7 See Goldberger (1964, 125) for the relevant asymptotic expression.

III / MAXIMUM LIKELIHOOD ESTIMATES OF THE LINEAR AND EXTENDED LINEAR EXPENDITURE SYSTEMS IN THE USA

The period 1930–72

The two models, LES and ELES, have been fitted to USA data on per capita expenditures, disposable income, and prices for the period 1930–72 (omitting 1942–6), using a five-commodity breakdown. Data sources, commodity definitions, and basic characteristics of the sample are given in the appendix.

First, each model was estimated on the assumption of no serial correlation, as specified in (8) (LES) and (9) (ELES). Estimated parameter values for β, γ, μ are given in Table 1 (columns headed ρ, $\lambda = 0$), together with estimates of their asymptotic standard errors, 't-ratios,' and measures of fit (R^2) and serial correlation (Durbin-Watson d-statistic).[8] The residuals on these equations exhibit strong positive autocorrelation, (see columns 9 and 10 in table 1). First-order transformations of the variables were then taken for both LES and ELES in the manner described in the previous section. A grid search was conducted for values of ρ and λ which maximized the likelihood function for each model.[9] Values at intervals of 0.1 in the range [0, 1] were used, narrowing to intervals of 0.05 near the maximum of the likelihood function. The likelihood functions were well behaved, and both reached a (unique) maximum when the autocorrelation coefficient was 0.85. The corresponding sets of estimates are given in Table 1 in columns headed $\rho,\lambda = 0.85$.[10]

The dominant features of the estimates presented in Table 1 are the following:

1 In each model, a common first-order autocorrelation coefficient removes the indications of strong serial correlation. Furthermore, the estimated value

8 For brevity we use 't-ratio' to describe the ratio of a parameter estimate to its standard error. Throughout the paper,

$$R^2 = 1 - (\sum_t \hat{u}_{it}^2 / \sum_t (v_{it} - \bar{v}_i)^2),$$

where v_i is the dependent variable, $\hat{u}_{it} = v_{it} - \hat{v}_{it}$, and the hat denotes estimated value. Since a 'least squares' criterion is not used in estimation, R^2 is to be interpreted solely as an index of the predictive power of the model. Similarly, the d-statistic is included merely to give an indication of patterns in the residuals.

9 Under both models, LES and ELES, the degree of autocorrelation in the residuals (as measured by the d-statistic) is very similar for all equations except Durables. It therefore seems a reasonable approximation to assume a common value of λ for each equation in ELES. A common ρ for the LES equations must be used for theoretical reasons associated with the singularity of the error variance-covariance matrix (see Berndt and Savin, forthcoming).

10 The R^2 values still refer to v_i and not its transform; the d values refer to the transformed equations, i.e. they relate to the residuals $\hat{u}_i^*$ and $\hat{e}_i^*$.

TABLE 1

Maximum likelihood estimates under alternative error assumptions: LES and ELES, USA 1930-72

	β				γ				R^2/d			
	$\rho, \lambda = 0$		$\rho, \lambda = 0.85$		$\rho, \lambda = 0$		$\rho, \lambda = 0.85$		$\rho, \lambda = 0$		$\rho, \lambda = 0.85$	
	LES (1)	ELES (2)	LES (3)	ELES (4)	LES (5)	ELES (6)	LES (7)	ELES (8)	LES (9)	ELES (10)	LES (11)	ELES (12)
Food	0.107	0.110	0.106	0.104	433.0	337.4	334.2	333.0	0.9948	0.9921	0.9995	0.9996
	(0.006)	(0.008)	(0.012)	(0.008)	(14.2)	(11.8)	(17.3)	(16.0)	0.22	0.25	1.82	1.92
	(19.0)	(14.2)	(9.2)	(13.8)	(30.5)	(28.5)	(19.3)	(20.8)				
Clothing	0.060	0.066	0.089	0.083	153.1	96.8	69.9	71.6	0.9909	0.9908	0.9985	0.9983
	(0.003)	(0.003)	(0.008)	(0.005)	(7.9)	(5.2)	(15.0)	(14.4)	0.22	0.31	2.08	2.01
	(20.2)	(19.2)	(11.1)	(15.4)	(19.2)	(18.6)	(4.7)	(5.0)				
Housing	0.202	0.178	0.131	0.159	224.8	80.4	92.4	96.0	0.9932	0.9920	0.9990	0.9987
	(0.006)	(0.004)	(0.009)	(0.007)	(25.4)	(12.6)	(22.1)	(21.3)	0.34	0.20	1.62	1.96
	(35.2)	(41.0)	(14.0)	(22.0)	(8.9)	(6.4)	(4.2)	(4.5)				
Durables	0.239	0.209	0.257	0.218	246.0	63.2	21.1	46.6	0.9952	0.9940	0.9947	0.9911
	(0.007)	(0.048)	(0.025)	(0.002)	(32.3)	(14.7)	(45.2)	(43.3)	1.28	1.60	2.28	2.37
	(36.8)	(43.9)	(10.2)	(13.6)	(7.6)	(4.3)	(0.5)	(1.1)				
Other	0.391	0.437	0.416	0.436	641.3	270.9	240.3	253.6	0.9990	0.9978	0.9996	0.9992
	(0.009)	(0.008)	(0.002)	(0.014)	(51.2)	(35.5)	(70.4)	(66.9)	0.46	0.36	2.27	2.11
	(44.2)	(54.5)	(20.8)	(30.5)	(12.5)	(7.6)	(3.4)	(3.8)				
Sum	1.000	1.000	1.000	1.000	1698.3	848.8	757.9	800.8				
					(130.5)	(72.2)	(150.6)	(144.6)				
					(13.0)	(11.8)	(5.0)	(5.5)				

NOTE: The estimated values of μ for ELES are 0.854 (0.008) (104.1) when $\lambda = 0$, and 0.863 (0.018) (47.8) when $\lambda = 0.85$. Period 1942-6 omitted. Standard errors are given in first parentheses, "t-values" in second. Dollars per capita in 1958 prices.

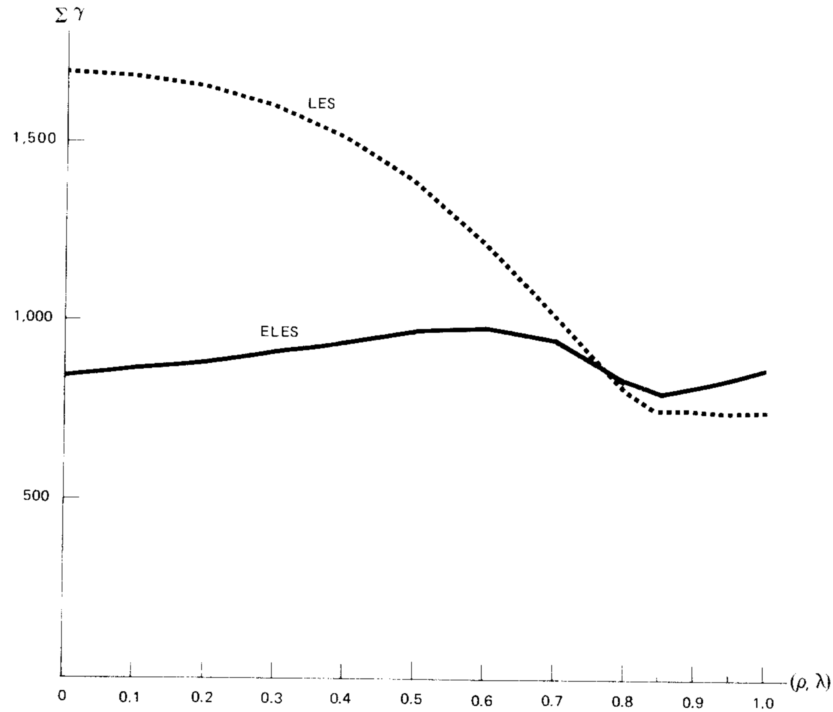

FIGURE 1 Estimates of $\Sigma\gamma$ for LES and ELES under alternative values of the autocorrelation coefficient, USA 1930–72

of the coefficient is the same in both models, $\rho = \lambda = 0.85$, and significantly different from zero at the 1 per cent level, applying the conventional likelihood ratio test (the χ^2 values on 1 degree of freedom are 176.4 and 205.8 for ρ and λ respectively).

2 LES estimates are very sensitive to error specification: the 'subsistence' estimates (the sum of the γ-estimates) go from \$1700 per head (at 1958 prices) for $\rho = 0$, to \$750 for $\rho = 0.85$. Only for $\rho > 0.7$ are the LES subsistence estimates consistent with the utility interpretation underlying model (8) i.e. only then is $v_t > \Sigma_i p_{it}\gamma_i$ for all t.[11] The relationship between the estimates $\Sigma\gamma$, ρ is depicted in Figure 1.

3 ELES estimates (in particular $\Sigma\gamma$) are not very sensitive to error specification. Also, they are consistent in all years with the utility interpretation underlying model (9) for all values of λ. The relationship between the estimates of $\Sigma\gamma$, λ in ELES is given in Figure 1.

4 Full maximum likelihood estimates of β,γ (estimates when $\rho = \gamma = 0.85$) are similar for both models. Precision of the β-estimates is higher in ELES; the same is true, but only marginally so, with the γ-estimates.

11 This feature of LES estimates has been noted before (Pollak and Wales, 1969; Lluch and Powell, forthcoming) and has provided impetus for applied work with shifting subsistence parameters (Pollak and Wales, 1969; Phlips, 1972).

TABLE 2
Mean expenditure and price elasticities: LES and ELES, USA 1930-72

	Expenditure	Price 1	2	3	4	5
Food						
LES	0.405	−0.275	−0.021	−0.026	−0.013	−0.071
ELES	0.422	−0.274	−0.023	−0.029	−0.019	−0.078
Clothing						
LES	0.899	−0.178	−0.478	−0.058	−0.029	−0.157
ELES	0.914	−0.182	−0.460	−0.062	−0.040	−0.170
Housing						
LES	1.125	−0.223	−0.058	−0.612	−0.036	−0.196
ELES	1.142	−0.228	−0.061	−0.591	−0.050	−0.212
Durables						
LES	1.612	−0.319	−0.084	−0.103	−0.825	−0.281
ELES	1.536	−0.306	−0.082	−0.104	−0.758	−0.285
Other						
LES	1.134	−0.225	−0.059	−0.073	−0.037	−0.742
ELES	1.142	−0.228	−0.061	−0.077	−0.050	−0.726

NOTE: Years 1942-6 omitted

In Table 2, we present expenditure and price elasticity estimates at mean sample values for both models, when $\rho = \lambda = 0.85$, as defined by (5). The similarity of parameter estimates noted above also yields similar elasticities. The sign patterns of the price elasticity matrices are consistent with the utility-maximization hypothesis, but users should be aware of potential underestimation due to rigidities of the model.[12]

The periods 1930–41, 1947–59, and 1960–72

It is very doubtful that linear Engel curves are an adequate representation of consumer behaviour over long time periods. Models (8) and (9) are indeed too rigid from this point of view, and it is important to try to assess the impact of non-linearities. The naïve way of doing so in this paper is to re-estimate ELES for short time periods. The rationale is that linear models over short periods represent linear segment approximations to a more complex underlying non-linear model.

12 Following standard practice, there is no distinction between durables and non-durables. The linearity of the model tends to reduce substitution possibilities. Phlips (1972, 455–6) reports short- and long-run elasticities estimated for eleven consumption categories over the period 1929–67 from a model with shifting parameters, under the Klein-Rubin utility specification. The long-run income elasticities, and the short-run uncompensated own price elasticities, upon rough aggregation of estimates over commodities, are broadly comparable to ELES estimates except for Housing.

By fitting ELES to subperiods, we expect to gain some information on the stability of estimated parameters over time. Such fitting is also advisable on common sense grounds: one wants to distinguish pre- and post-war economic conditions; and for predictions outside the sample period, one wants parameter values relevant for 'current' economic conditions.

Model (9), modified to incorporate first-order autocorrelation in the errors, has been fitted to three subperiods: 1930–41, 1947–59, 1960–72. The parameter estimates are given in Table 3. The dominant features of these results are as follows:

1 The estimated 35 per cent increase in 'subsistence' expenditure between 1930–41 and 1960–72;[13]

2 The substantial reduction in the Food marginal budget shares, from 0.29 to 0.09, coupled with increases in β (Housing) – from 0.05 to 0.16 – and β (Other) – from 0.29 to 0.43;

3 The estimated increase in the MPC, from 0.64 in the thirties to 0.81 in the sixties;

4 The change over time in the pattern of serial correlation, as measured by the estimated value of the first-order autocorrelation coefficient, which is substantially smaller in the most recent subperiod (and not significantly different from zero at the 5 per cent level).

In summary: the ELES fit to subperiods is indeed called for, on account of rather substantial shifts in demand and saving parameters over time.

IV / THE ELES AGGREGATE CONSUMPTION FUNCTION

The aggregate consumption function associated with ELES is given by equation (10). If the error terms e_i on the individual expenditure equations are not autocorrelated, then the errors on the aggregate consumption function are independently distributed with common variance $\sigma^2 = \iota'\Omega\iota$, where ι is the n-vector of unit elements. If the variables of the expenditure equations are transformed using the first-order autocorrelation coefficient λ, corresponding transformations of the variables in the associated aggregate consumption function will ensure that the new error terms e_t^* are uncorrelated over time with common variance $\sigma^{*2} = \iota'\Omega^*\iota$.

Equation (10) can be looked upon as a stochastic specification of a naïve Keynesian consumption function with a variable intercept, defined as a linear combination of prices. In fact, absence of relative price changes over the sample period in (10) yields a Keynesian model. If $p_{it} = P_t$,

13 ELES λ-estimates for each subperiod were again much less sensitive to the specification of autocorrelation in the errors than were the corresponding LES estimates. This was particularly so for the period in which autocorrelation was most apparent, namely, 1947–59.

TABLE 3

Maximum likelihood estimates for subperiods, ELES, USA

	1930-41			1947-59			1960-72		
	β	γ	R^2/d	β	γ	R^2/d	β	γ	R^2/d
Food	0.286 (0.023) (12.5)	283.2 (16.6) (17.1)	0.9784 1.45	0.157 (0.032) (4.9)	316.2 (26.4) (12.0)	0.9778 2.11	0.088 (0.005) (17.4)	397.5 (13.9) (28.5)	0.9987 2.03
Clothing	0.109 (0.012) (9.2)	105.1 (5.3) (20.0)	0.9759 2.14	0.082 (0.016) (5.1)	80.2 (19.5) (4.1)	0.8849 1.96	0.088 (0.004) (19.6)	101.3 (17.0) (5.9)	0.9952 2.08
Housing	0.051 (0.007) (7.3)	106.7 (1.5) (72.6)	0.9939 1.23	0.063 (0.018) (3.6)	207.2 (12.7) (16.3)	0.9956 1.45	0.162 (0.006) (26.8)	167.5 (23.3) (7.2)	0.9991 1.67
Durables	0.265 (0.022) (12.3)	62.9 (9.1) (6.9)	0.9752 1.72	0.403 (0.083) (4.8)	−38.6 (151.8) (0.3)	0.7119 1.89	0.228 (0.008) (28.9)	133.0 (46.4) (2.9)	0.9789 1.84
Other	0.288 (0.013) (21.8)	355.0 (11.9) (29.8)	0.9859 1.32	0.294 (0.041) (7.1)	439.3 (60.2) (7.3)	0.9960 1.25	0.434 (0.012) (35.4)	435.0 (74.3) (5.9)	0.9979 1.68
Sum	1.000	912.9 (40.9) (22.3)		1.000	1004.4 (247.1) (4.1)		1.000	1234.3 (172.0) (7.2)	
μ	0.639 (0.041) (15.6)			0.809 (0.071) (11.4)			0.814 (0.027) (30.3)		
λ	0.6			0.9			0.3		
χ_1^2	24.62			48.76			3.26		

NOTE: Standard errors are given in first parentheses; "t-values" in second. R^2 values relate to original variables, d values refer to transformed variables. γ is in dollars per capita in 1958 prices.

where P_t is an index of general consumer prices, substitution into (10) yields

$$(v_t/P_t) = \alpha + \mu(y_t/P_t) + (\epsilon_t/P_t), \tag{13}$$

where

$$\alpha = (1 - \mu) \sum_{i=1}^{n} \gamma_i.$$

The interpretation of (10) as a naïve Keynesian consumption function is particularly appropriate in this paper, given the assumed equality of measured and permanent income.

It is of interest to ascertain whether objections against the naïve Keynesian consumption function (see Evans, 1969, 15–17) are removed by the introduction of relative prices (but not permanent income). Notice, in particular, that the 'variable intercept' in (10) implies that the average propensity to consume (APC) does not necessarily decline as real income increases, as it must in model (13). Changes in relative prices might conceivably compensate for the increase in real income. In particular, increases in the relative prices of commodities with 'large' subsistence components (the γ-estimates) might produce an increase in the APC even when real income increases.

For the purposes of comparison, consider

$$\hat{v}_t/P_t = \hat{\alpha}_t + \hat{\mu}(y_t/P_t), \quad \hat{\alpha}_t = (1 - \mu)(\Sigma p_{it}\gamma_i)/P_t; \tag{14a}$$

$$\tilde{v}_t/P_t = \tilde{\alpha} + \tilde{\mu}(y_t/P_t), \tag{14b}$$

where P_t is the implicit deflator for consumption expenditures, 1958 = 1.000. Equation (14*a*) is the fitted ELES consumption function in 'real' terms using μ, γ estimates for model (9) (after transforming to allow for first-order autocorrelation in the errors). Equation (14*b*) is the naïve Keynesian consumption function estimated by the Cochrane-Orcutt procedure. Both equations have been fitted for the period 1930–72 and three subperiods (the fit to subperiods being an approximation to a non-linear relationship). Equation estimates with the ranges and means of α_t for (14*a*) are given in Table 4.

The general conclusion reached is that the ELES and naïve Keynesian aggregate consumption functions yield very similar results. Essentially this is because variations in relative commodity prices over the period 1930–72 have been comparatively small.[14] Under these conditions ELES reduces to the Keynesian model. Changes in the intercept term between subperiods are then explained in terms of changes in μ and γ, the MPC and the 'subsistence' parameters respectively.

14 At least for the level of commodity aggregation used here

TABLE 4

Aggregate consumption functions, USA

	1930-72[1]	1930-41	1947-59	1960-72
Keynesian				
$\tilde{\alpha}$	90.3	337.8	171.8	153.3
	(17.6)	(48.3)	(79.8)	(34.7)
$\tilde{\mu}$	0.869	0.641	0.819	0.844
	(0.010)	(0.042)	(0.046)	(0.015)
R^2	0.9975	0.9677	0.9781	0.9959
d	1.43	0.70	1.65	2.16
$\tilde{\lambda}$	0.278	0.453	0.160	−0.079
ELES				
$\Sigma\hat{\alpha}_t/T$	110.0	330.3	189.5	230.5
	(14.0)	(38.3)	(73.5)	(35.6)
$\hat{\mu}$	0.863	0.639	0.809	0.814
	(0.018)	(0.041)	(0.071)	(0.027)
R^2	0.9970	0.9567	0.9784	0.9942
d	1.17	0.49	1.63	1.52
$\hat{\lambda}$	0.85	0.60	0.90	0.30
min $\hat{\alpha}_t$	107.8	328.3	186.1	229.1
(year)	(1932)	(1933)	(1947)	(1960)
max $\hat{\alpha}_t$	112.2	331.5	191.9	232.8
(year)	(1972)	(1939)	(1955)	(1972)

NOTE: Standard errors are given in parentheses. For each model, the parameter estimates and standard errors are derived under the assumption of first-order autocorrelation in the errors. The R^2 and d values are calculated using these parameter estimates and *non-transformed* variables, where the dependent variable is v/P. In the Keynesian model, the first-order autoregressive coefficient $\tilde{\lambda}$ used to transform the data is calculated as $1-\bar{d}/2$, where $\bar{d}$ is the OLS estimate.

1 Years 1942-46 omitted. Durbin-Watson statistics are adjusted for the gap by using (residual 1947 − residual 1946) in the numerator.

For the long period, the residuals $v_t - \hat{v}_t$ for the ELES consumption function average − 6.86[15] (equivalent to 0.5 per cent of mean total consumption in current prices) and are negative in twenty-three out of the thirty-eight years. Thus the model tends to overestimate consumption, or equivalently, underestimate savings (the mean residual represents 5.1 per cent of mean savings).

V / CONCLUSIONS

In this paper it has been shown that a great deal of the well known variation in the γ-estimates of LES can be attributed to error specification – in

15 The residuals from the ELES subperiod equations imply net overestimation of consumption in 1947–59 and 1960–72, underestimation in 1930–41.

TABLE 5

Basic characteristics of the sample

	p_1	p_2	p_3	p_4	p_5	P	w_1	w_2	w_3	w_4	w_5	v	y
1930	0.501	0.488	0.791	0.558	0.499	0.535	0.258	0.115	0.158	0.103	0.367	566.6	604.8
1935	0.433	0.408	0.558	0.444	0.433	0.445	0.290	0.108	0.138	0.093	0.317	438.0	459.3
1941	0.458	0.461	0.630	0.508	0.477	0.488	0.290	0.109	0.127	0.120	0.354	604.2	694.7
1947	0.837	0.900	0.704	0.826	0.704	0.779	0.325	0.117	0.098	0.127	0.333	1115.2	1178.6
1953	0.942	0.965	0.907	0.943	0.882	0.918	0.280	0.096	0.127	0.144	0.352	1441.1	1582.5
1959	0.987	1.011	1.019	1.014	1.030	1.013	0.252	0.085	0.140	0.142	0.380	1757.8	1904.7
1965	1.073	1.075	1.093	0.995	1.142	1.089	0.228	0.083	0.147	0.153	0.389	2228.0	2435.6
1972	1.375	1.435	1.360	1.128	1.509	1.379	0.200	0.086	0.145	0.162	0.407	3479.0	3816.8

particular, the value of the first-order autocorrelation coefficient. An alternative model with income as the explanatory variable, ELES, is shown to yield estimates which are more stable. Estimates from both models are roughly similar (as they should be) for values of the coefficient which maximize the likelihood function in each model.

The ELES model contains additional information (estimates of the MPC) at no additional cost. It has been used to investigate shifts in demand and savings patterns over three subperiods. Also, a comparison of the ELES aggregate consumption function with a naïve Keynesian one is given. The introduction of relative prices – with the γ-parameters – in the intercept of such a function does not eliminate the well known deficiences of using only current income as the explanatory variable. Extensions to include permanent income are called for. These, together with model changes to allow for additional substitution possibilities, represent useful directions of work.

APPENDIX

Data sources
The expenditure series, in both constant and current dollars, disposable personal income, and population were obtained from *The National Income and Product Accounts of the United States, 1929–1965, Statistical Tables* and July issues of the *Survey of Current Business*. The price variables are implicit deflators (1958 = 1.000).

Commodity definitions
The five categories of goods used were aggregated from the basic eleven-good breakdown as follows: 1/ Food: Food and Beverages; 2/ Clothing: Clothing and Shoes; 3/ Housing: Housing; 4/ Durables: Automobiles and Parts, Furniture and Household Equipment, and Other Durable; 5/ Other: Gasoline and Oil, Other Non-Durable, Household Operation, Transportation, and Other Services.

The basic characteristics of the sample are given in Table 5.

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